Closingt Wed: HW_4A, 4B, 4C (6.4, 6.5)
Midterm 1 will be returned Tuesday
Grades will be posted by the end of next week
See all of my posted materials on 6.4!

1. Compilation of old midterm questions by me with full solutions (I suggest you look through this before you do the homework).
2. Compilation of many, many old final questions with full solutions.

<u>6.4 Work</u>

Work = "total effort"

In other words, it is measure of energy expended in completing a task.
Work is a fundamental concept in physics and engineering.

When a **constant** force is applied through a fixed distance, we define

Work = Force · Distance
W = F · D

If force or distance change in some way during the task (i.e. NOT constant), then we can use calculus to break up the problem into subtasks, approximate with F · D on each subtask, and add up the approximations. That is,

$$\text{Work} = \lim_{n\to\infty} \sum_{i=1}^{n} (\text{Force} \ \cdot \ \text{Distance})$$

But to use calculus, our job is to break up into subdivisions and find the pattern for the force and the distance for each subdivision.

First, some units:

Recall Newton's second law

Force = Mass· Acceleration

$F = m \cdot a$

	Metric	Standard
Mass		
Acceleration		
Force		
Distance		
Work		

PROBLEM TYPE 1: *Force changing*.
(Leaky buckets and springs)
If we are moving an object from
$x = a$ to $x = b$ and $f(x)$ = "force at x",
then

$$\text{Work} = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \int_a^b f(x)dx$$

Examples:

1. *Leaky bucket*: A leaking bucket is lifted 10 feet. At the bottom the bucket weighs 120 pounds and at the top the bucket weighs 100 pounds. Assume the water leaked out a constant (linear) rate as it was lifted. How much work was done to life the bucket?

2. *Springs*:

A weight is attached to the end of a spring and the other end is attached to the wall. We label x = 0 to the location when where the weight is at rest (natural length).

Hooke's law: Force is proportional to the distance from natural length.
If x = distance from natural length,
Then $f(x) = kx$, for some constant k.

Example: Assume natural length for a given spring is 5 cm from the wall.
And you know 5 Joules of work are done to stretch from 5cm from wall to 9cm from wall. How much work is done to stretch from 7cm to 10cm from wall?

PROBLEM TYPE 2:

Force and dist. changing.

(Chains and pumping)

In some problems, we subdivide the task and find

d(x) = `dist. for subtask starting at *x*'

f(x) = `density of subtask at *x*'

f(x)Δx = `force of subtask at *x*'

in which case:

$$\text{Work} = \lim_{n\to\infty} \sum_{i=1}^{n} d(x_i) f(x_i) \Delta x = \int_a^b d(x) f(x) dx$$

Example:

1. You are lifting a heavy chain to the top of a building. The chain has a density of 3 lbs/foot. The chain hangs over the side by 25 feet before you start pulling it up. How much work is done in pulling the chain all the way to the top?

2. You are pumping water out of a tank. The tank is a rectangular box with a base of 2 ft by 3 ft and height of 10ft. The density of water is 62.5 lbs/ft^3. If the tank starts full, how much work is done in pumping all the water to the top and out over the side?